Applied cryptography with quantum, post-quantum and traditional insights. A popularisation talk

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Bruno Martin

I3S Laboratory (CS), Université Côte d'Azur

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Quizz

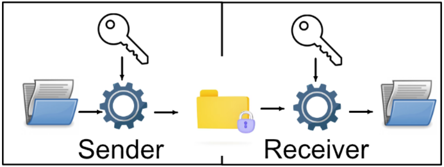
In current Internet secured protocols (`https`, `gpg`, `S/MIME`), do you think the data is encrypted with:

- ☐ Secret Key
- ☐ Public Key
- ☐ Other

Correct answer:

Hybrid Encryption (Other)

Secret Key Cryptography



- Stream cipher (Vernam) ensures perfect security (Information theoretic)
- Blocks chaining encryption (AES-256-CTR) ensures semantic security (complexity theoretic)

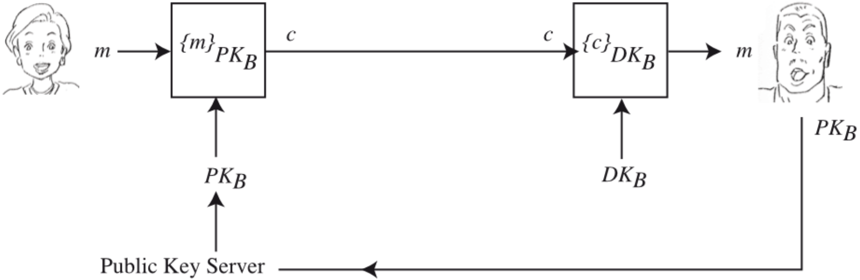
Pros:

Cleartext and Ciphertext are about the sime size ; quick computation

Cons:

Secret Key transmission

Public Key Cryptography



Basically RSA encryption (with padding schemes) or ciphers based on number theory problems (factoring, discrete log.)

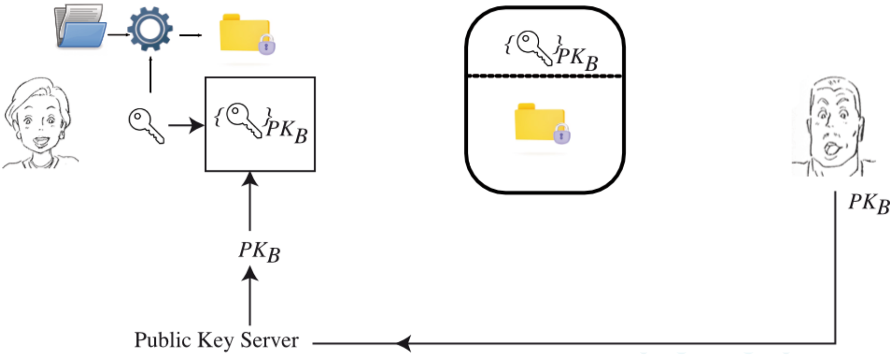
Pros:

Public key transmission

Cons:

Slow computation (factor 4k); Ciphertext's size larger than cleartext

Hybrid Encryption



Pros:

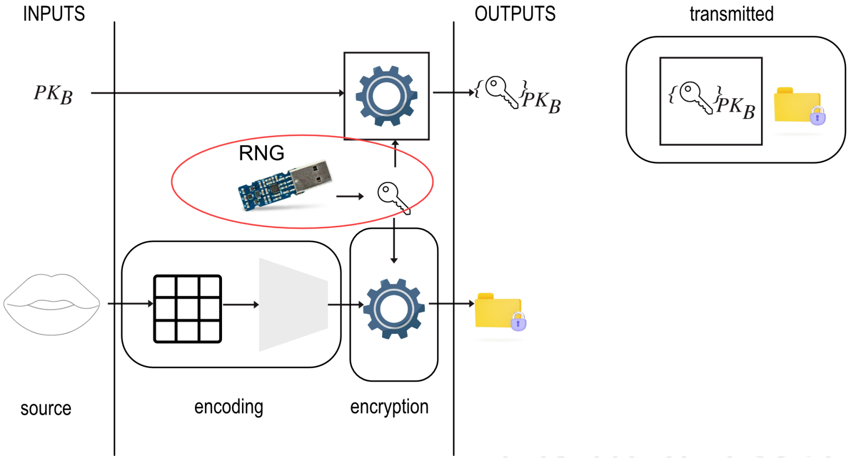
Public key transmission; Cleartext and ciphertext the same size; quick computation

Cons:

Not quantum safe. . . (Wait a bit)

Complete processing chain

Focus on RNG



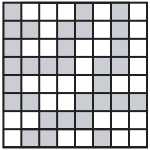
See [Krasnowski, 2021]'s PhD co-advised with J. Lebrun (Signal processing) for a complete processing chain

Random Number Generation

- TRNG: uses a nondeterministic source to make randomness. Random numbers come from measuring unpredictable natural processes (pulse detectors of ionizing radiation activities, gas discharge tubes, and leaky capacitors,...).
- QRNG: exploit elementary quantum optic processes that are intrinsically probablilistic to generate true randomness. Random numbers are a result of measurement on a quantum system.
- PRNG: runs an algorithm that uses mathematical formulas or algorithms to produce random numbers.

Random Generator (rule 30) – Example of RNG

t=0



[Wolfram, 1986]: given i, $\{x_i^t\}_{t \geq 0}$ is pseudo-random.
Used in Mathematica™.
Justified by Knuth's statistical tests.
Not suitable for cryptography; can be improved [Martin et al., 2014]

What is a binary random sequence?

A random sequence

- is unpredictable
- is uncompressible (there is no shorter program than the program which prints out the random sequence)
- passes all (effective) statistical tests

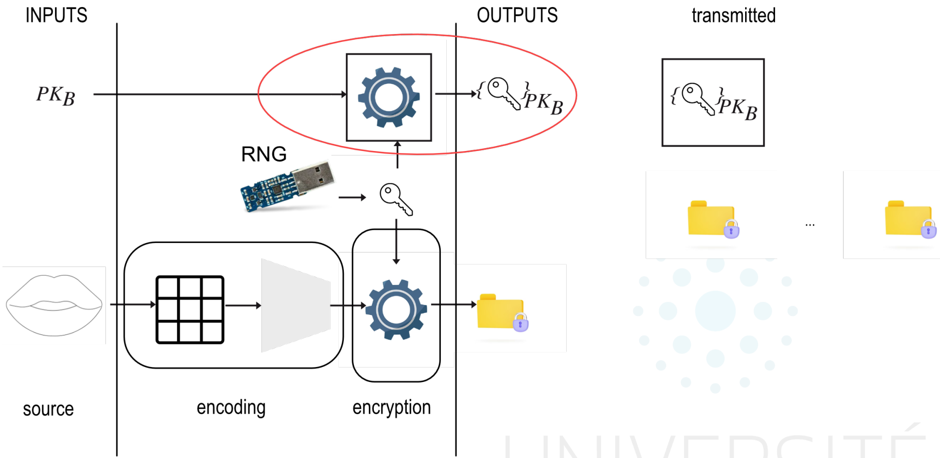
No program can generate a true random sequence, only pseudo-random. Random sequences are obtained by observing natural phenomenon.

Randomness definitions

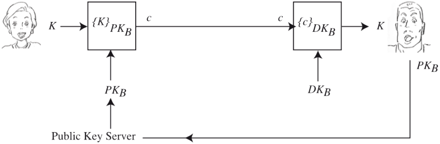
- TRS: a sequence that is unpredictable
- PRS: a sequence that cannot be distinguished from a TRS by any PPT algorithm.

Complete processing chain

Focus on Key Transportation

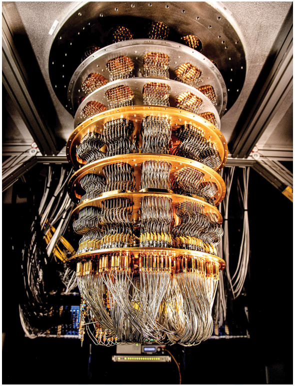


Key Transportation



- Today's PKC: RSA, DH, ECDH
- Based on number theoretic problems
- Increased importance of ECC

(co-advisor with A. Hirschowitz of a PhD on ECC [Virat, 2009])



- Shor's algorithm in QP
- Simon's Algo in QP
- 2300 qubits to break RSA-1024
- IBM Osprey: 433 qubits

HSP, Simon, Shor

Definition

Given G a group, $H \leq G$ a subgroup, X a finite set, $f : G \to X$ hides H if, $\forall g_1, g_2 \in G$, $f(g_1) = f(g_2)$ iff $g_1 H = g_2 H$.

Hidden Subgroup Problem

For a group G, X a finite set, $f : G \to X$ hides $H \leq G$.
Given f by an oracle using $O(\log |G| + \log |X|)$ bits and using evaluations of f via its oracle, determine a generating set for H

- [Simon, 1997] exhibited a quantum algorithm that solves Simon's problem (a special case of HSP)
- [Shor, 1999]'s quantum algorithm for factoring and discrete logarithm computing relies on the ability of quantum computers to solve the HSP for finite Abelian groups.

Going Post-Quantum

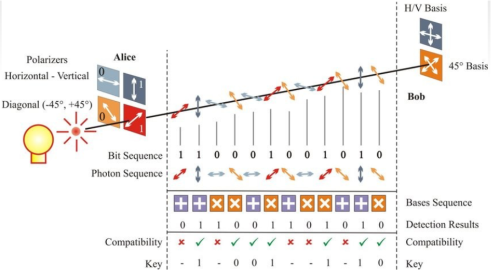
Replace traditional PKC

- Shor's and Simon's algorithms solve in quantum-polynomial time:
  - Integer factorization. RSA is dead.
  - The discrete-logarithm problem in finite fields[1]. DSA is dead
  - The discrete-logarithm problem on elliptic curves. ECDH is dead
- **Post-quantum crypto** must resist attacks by quantum computers
- Replace RSA, DSA, ECDH by new standards
- Current standards (2022) rely on the problem **Learning With Errors** over arithmetic lattices.
  - CRYSTALS-Kyber for encryption (keysizes: pk=1184, dk=2400, block=1088)
  - CRYSTALS-Dilithium for signatures
- In use: OpenSSH, Cloudflare, AWS, IBM backup device

[1] DLOG computation requires half the number of qubits required to factor an integer of the same size

Going Quantum

Remove PKC



Key transportation with [Bennett and Brassard, 1984] or [Ekert, 1991]. Nice survey [Pirandola et al., 2020] (approx. 200p)

Pros:

Highly secure

Cons:

Slow throughput; relatively small distance; requires two channels

Goal achieved

Key transportation

Different ways to transport a secret key.
Either with

- PQC
- QKD

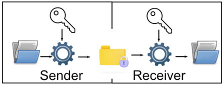
First step

Alice and Bob share a key !

They can use it to encipher a message

Secret Key Cryptography

Stream cipher (Vernam)



A and B share a random sequence of n bits: the secret key K.
A enciphers M of n bits in $C = M \oplus K$. B deciphers C in $M = K \oplus C$.

Example

$M = 0011, K = 0101$
$C = 0011 \oplus 0101 = 0110$
$M = K \oplus C$.

Pros:

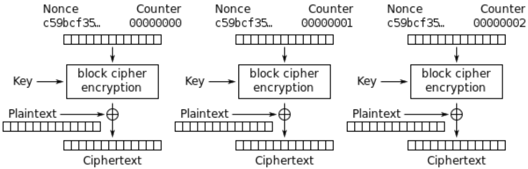
Quick ; high throughput ; perfectly secure

Cons:

Long and perfectly random key ; not reusable

Secret Key Cryptography

Blocks chaining encryption (AES-256-CTR)



Counter (CTR) mode encryption

Pros:

Quick ; high throughput ; short key ; semantic security ; quantum safe

Cons:

not perfectly secure

Quantum attacks against SKC

Searching the key uses [Simon, 1997]'s or [Grover, 1996]'s algorithms.

Grover's algorithm

Search an element among n items requires time $n/2$ on the average or time n in the worst case with a classical computer. It can be done in $\sqrt{n}$ steps on a quantum computer.

Pros:

Up to 4 qubits required (for Grover)

Cons:

Exponential algorithm (square root speedup compared with brute-force.)

Security Notions

- **Perfect security** is about confidentiality against arbitrary adversaries. It is based on information theory. It can be achieved with Vernam Cipher with a TRNG or QRNG
- **Semantic security** is about confidentiality against computationally bounded adversaries. It is based on complexity theory and the adversary is a PPT algorithm. It can be achieved with PRNG
- **Quantum safe** is about confidentiality against computationally bounded adversaries. It is based on complexity theory and the adversary is a quantum algorithm

Goal achieved

Encryption

Different encryptions to secure a message

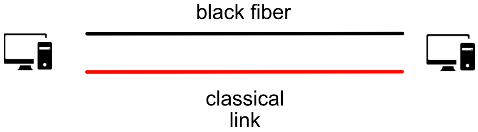
- Vernam cipher to achieve perfect security
- Traditionnal ciphers to achieve quantum safety (with a key large enough at least 256 bits)

Second step

Alice and Bob can communicate securely !

Going further

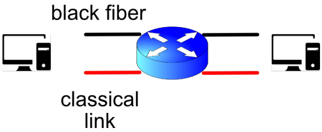
Direct connection



- When both quantum and classical links are available (150km).
- QKD can be achieved and the key used to encipher data (with perfect security or quantum safety)

Going further

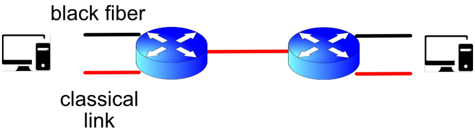
Indirect connection – 1 hop



- When both quantum and classical links are available between end systems interconnected with a single router
- QKD can be achieved between end systems and the router
- A protocol has to be designed to generate and transport a key
- Quantum safe encryption can be achieved (or better ?)

Going further

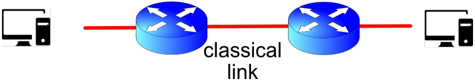
Indirect connection – many hops



- When both quantum and classical links are available between end systems and routers interconnected with a classical link
- QKD can be achieved between end systems and routers but not inbetween.
- A protocol has to be designed to generate and transport a key
- Quantum safe encryption can be achieved

Going further

Classical link



- When no quantum links are available
- Key transportation has to be done with post-quantum cryptography
- Quantum safe encryption can be achieved

Integration

We intend to integrate the previous cases in standard librairies to secure

- IP layer with IPSec
- TCP layer with TLS, which ensures security of classical Internet protocols (`http`, `smtp`, `imap`,...)

with different levels of security.

Thanks for your attention

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