Computing Tree-decompositions of Graphs

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Graph decompositions and treewidth. *Tree-decompositions* of graphs are a way to decompose a graph into small "pieces" (called bags) with a tree shape. To every tree-decomposition of a graph a measure is associated (its *width*). In the last decades, graph decompositions have been studied a lot for their algorithmic applications (among others, they are an important ingredient of the celebrated Robertson and Seymour work on Graph Minors). Indeed, once a graph admits a good decomposition (the smaller the width is, the better the decomposition is), many hard problems can be solved efficiently using dynamic programming (e.g., the famous Courcelle's Theorem). Unfortunately, computing a good decomposition is a hard task by itself. More precisely, computing an optimal tree-decomposition of a graph is an NP-hard problem. There are algorithms that decide if a graph has *treewidth* at most k in polynomial time (for a fixed k) but none of them is practical even for small graphs. Therefore, a research effort has to be done to find efficient algorithms to compute tree-decompositions.

One way to tackle this question is to focus on different measures of tree-decompositions [KLNS15, Seymour16]. For instance, the *treelength* (measuring the diameter of the bags) [DG07,CDN16] and the *treebreadth* (corresponding to the radius of the bags) are known to be NP-hard to compute but admit efficient approximation algorithms in general graphs (e.g., [DLN16]). Moreover, the treelength and the treewidth are equivalent (up to a constant ratio) in large graph classes [CDN16]. For these reasons, it is important to investigate the computational complexity of treebreadth in such graphs.

Objectives of the TER. The ambitious objectives of the TER may be some of the following:

- Study the computational complexity of treelength (or treebreadth) in the class of planar graphs.
- Propose better approximation algorithms for treelength or treebreadth in general graphs.
- Turn the result of [CDN14] into an algorithmical one (i.e., find an algorithm that, given a tree-decomposition with small length, computes one with small width).

This TER may be followed by an internship (and then, by a Ph.D. thesis).

References:

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KLNS15 Adrian Kosowski, Bi Li, Nicolas Nisse, Karol Suchan: k-Chordal Graphs: From Cops and Robber to Compact Routing via Treewidth. Algorithmica 72(3): 758-777 (2015)

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