Vérification de programmes

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Interprétation Abstraite

- Interprétation Abstraite (IA) est une théorie des approximations de sémantiques [Cousot and Cousot, 1976]
- Utilisée pour l'analyse statique et la vérification des logiciels
- Exemple d'application : prouver automatiquement qu'un programme ne contient pas d'erreurs d'exécution

Interprétation Abstraite

Étudie les valeurs des variables

```
y ← 1
x ← random(1, 5)
while x≤8 and y<3 do
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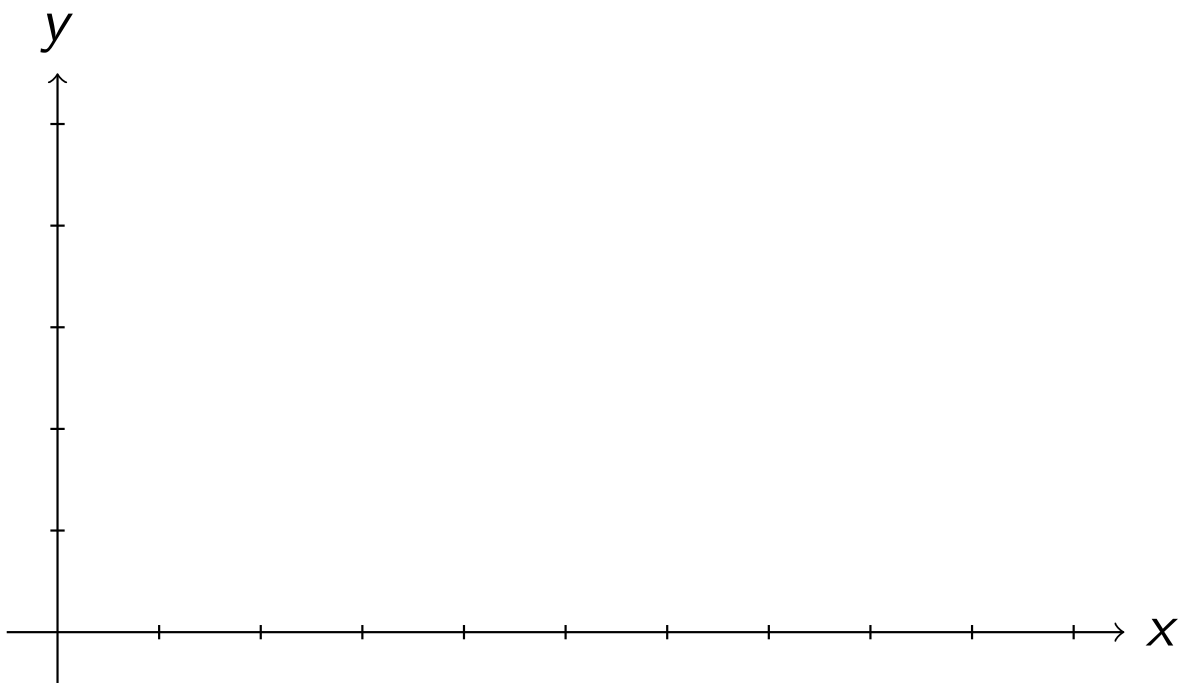

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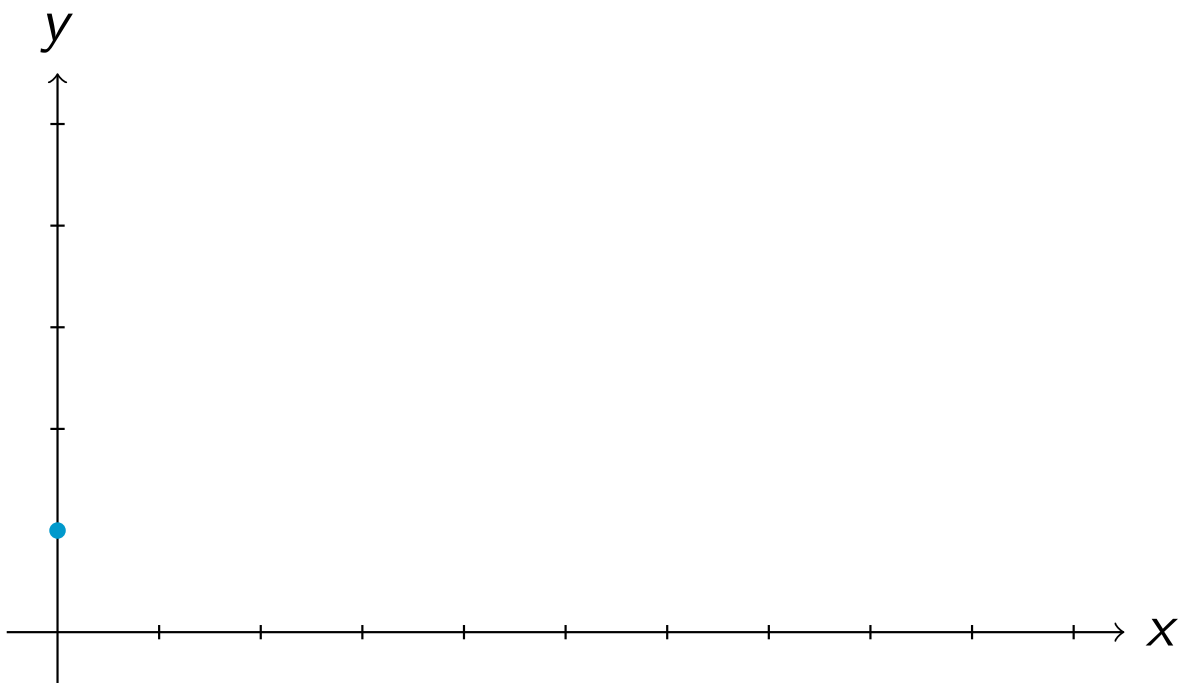

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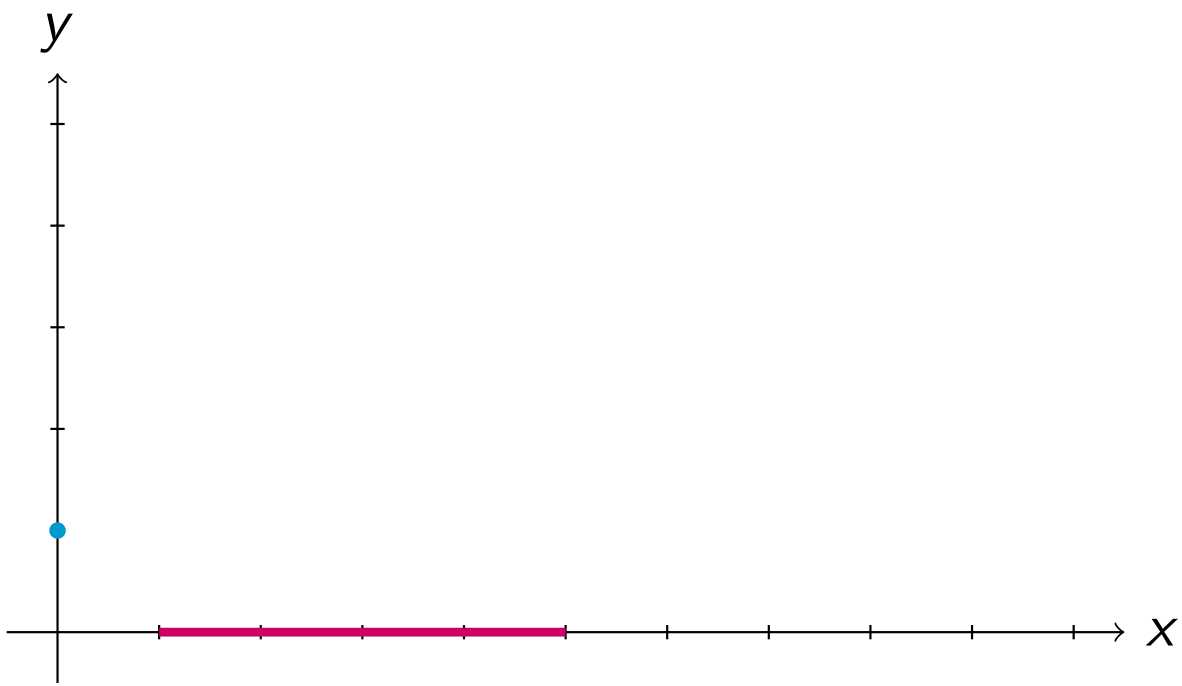

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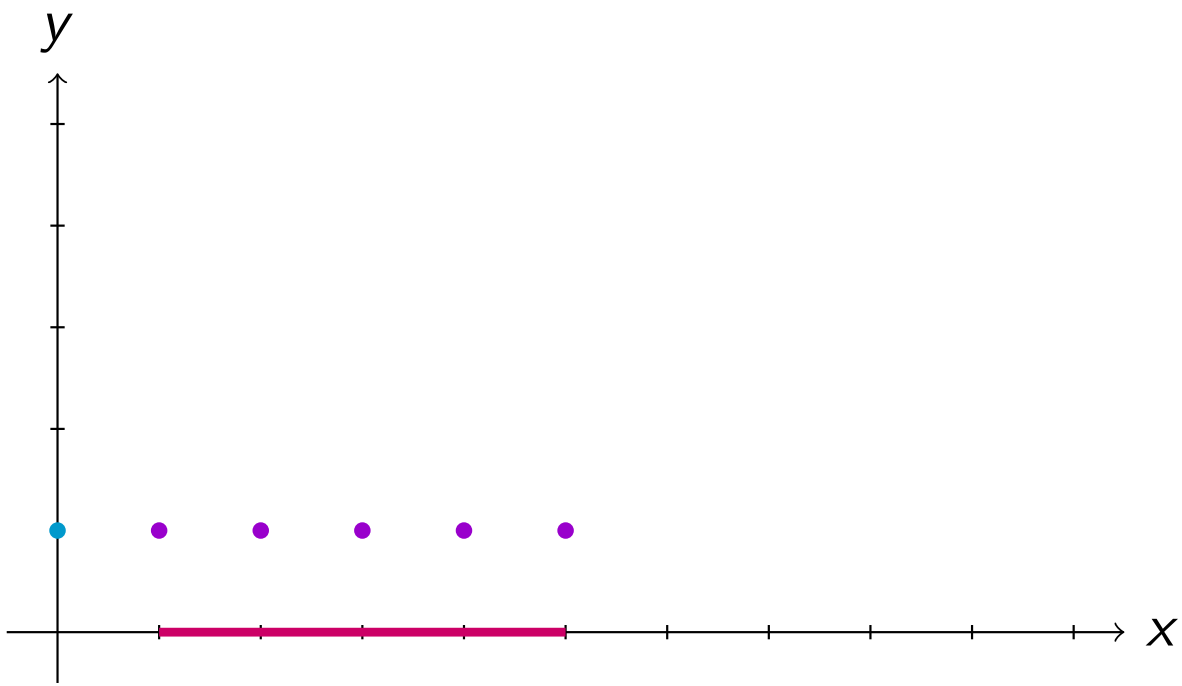

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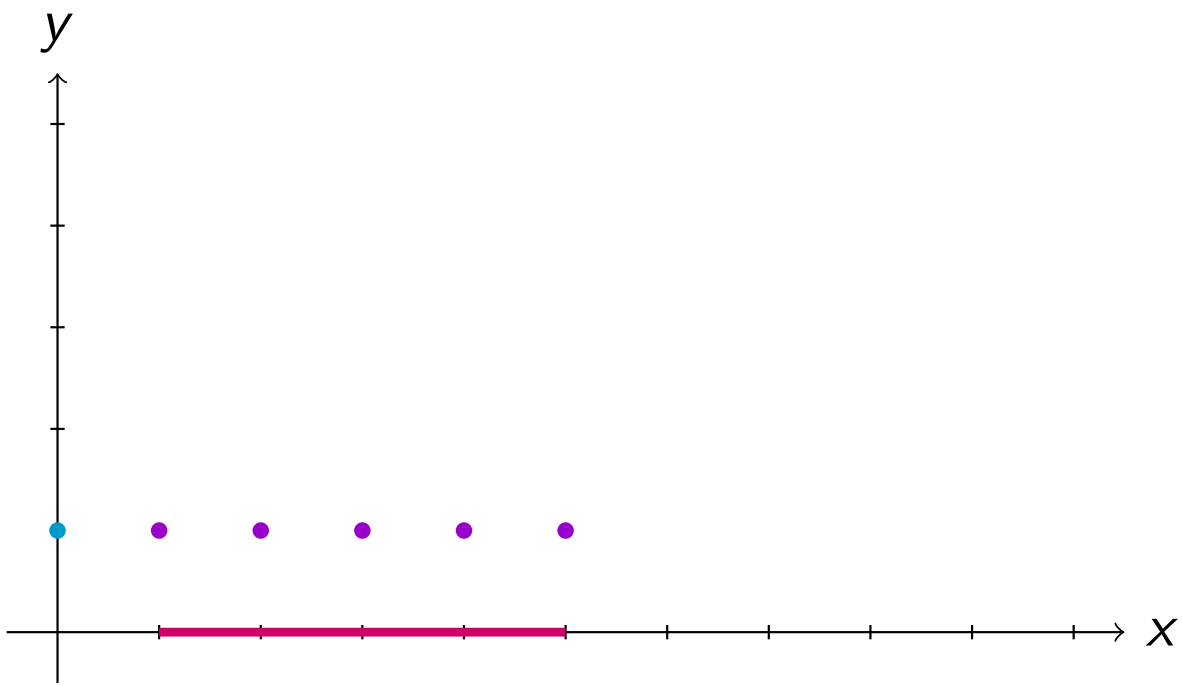

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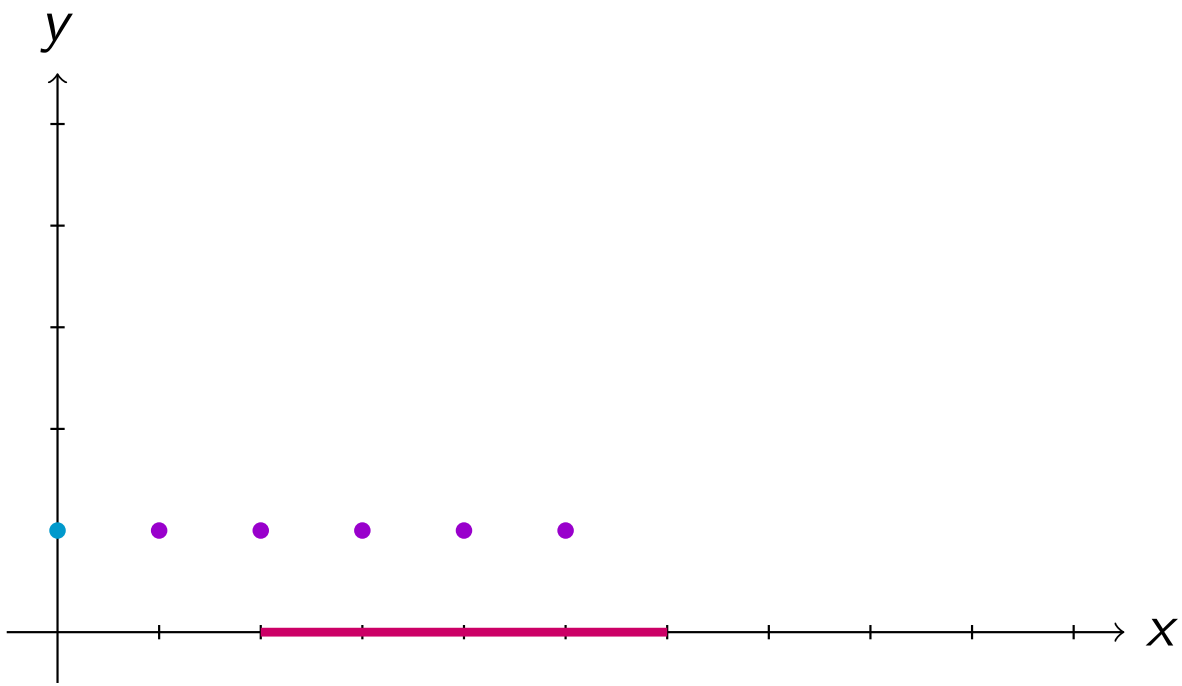

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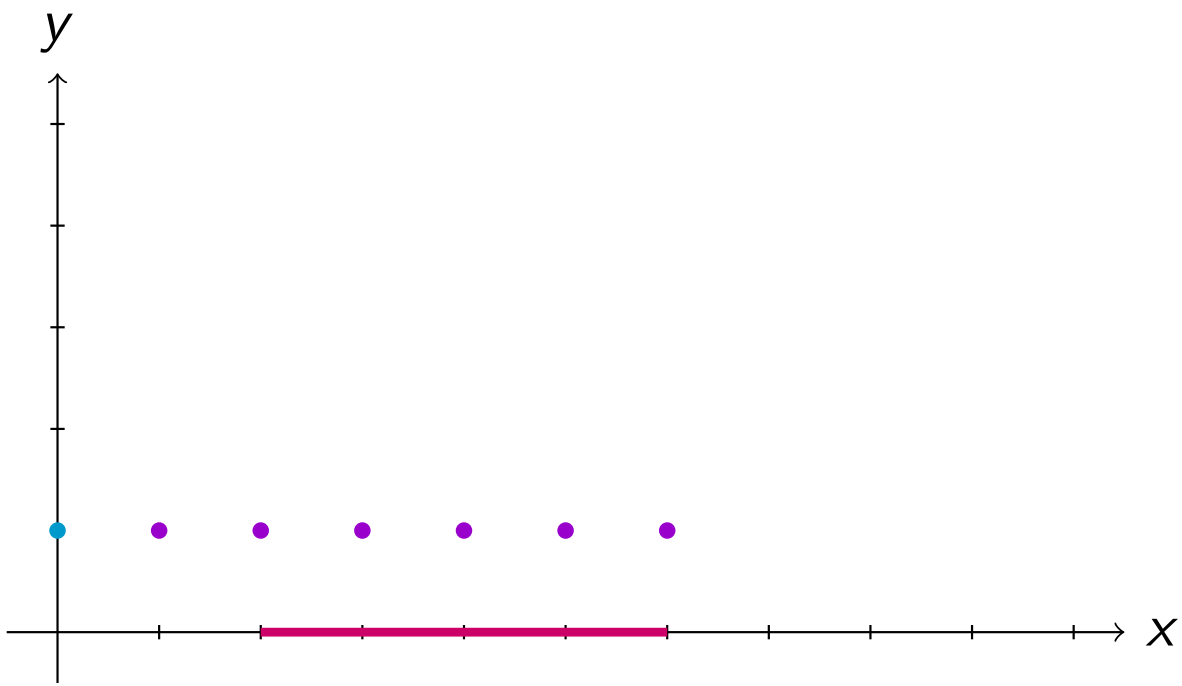

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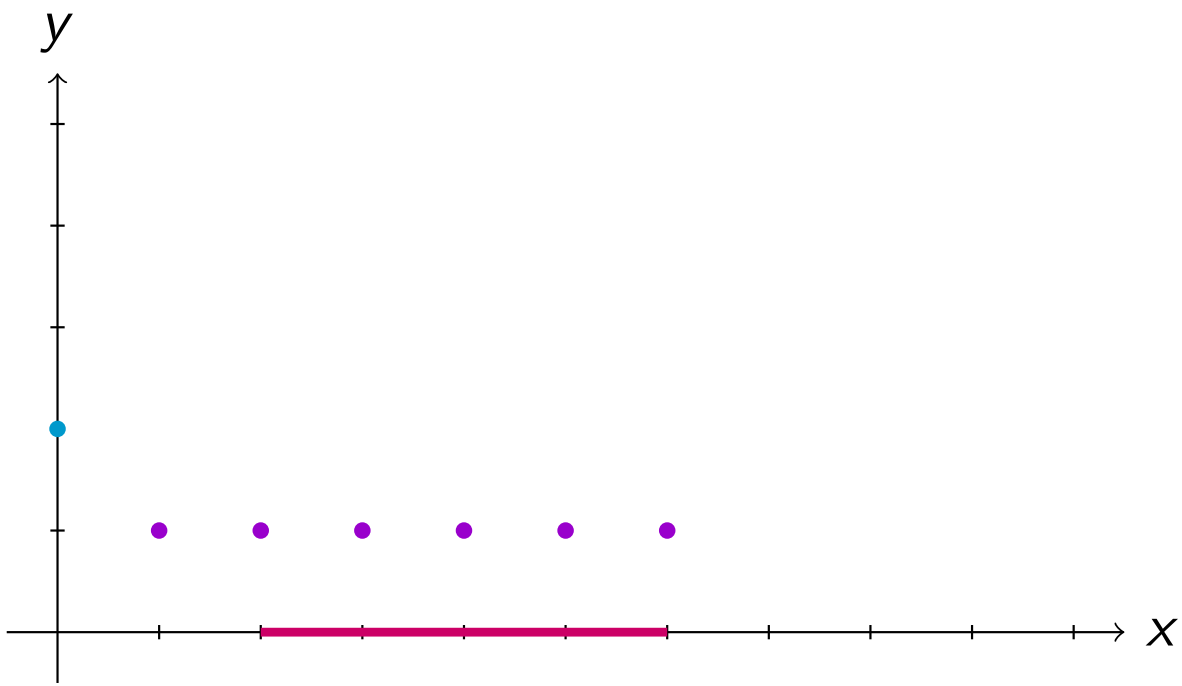

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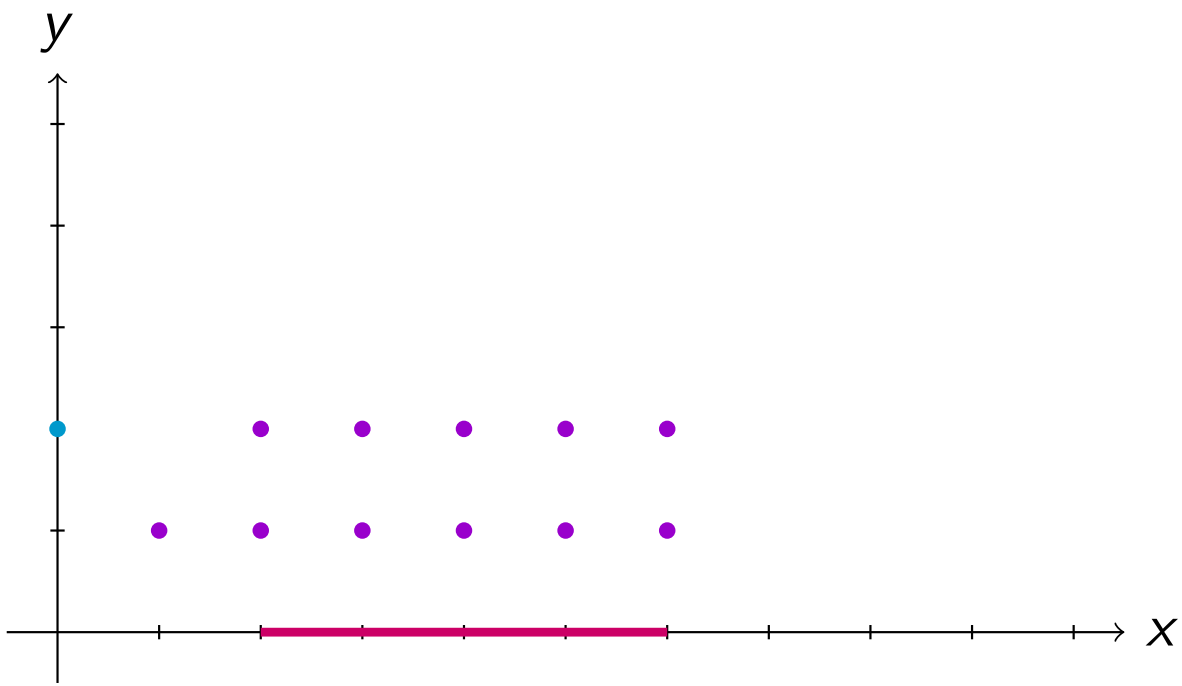

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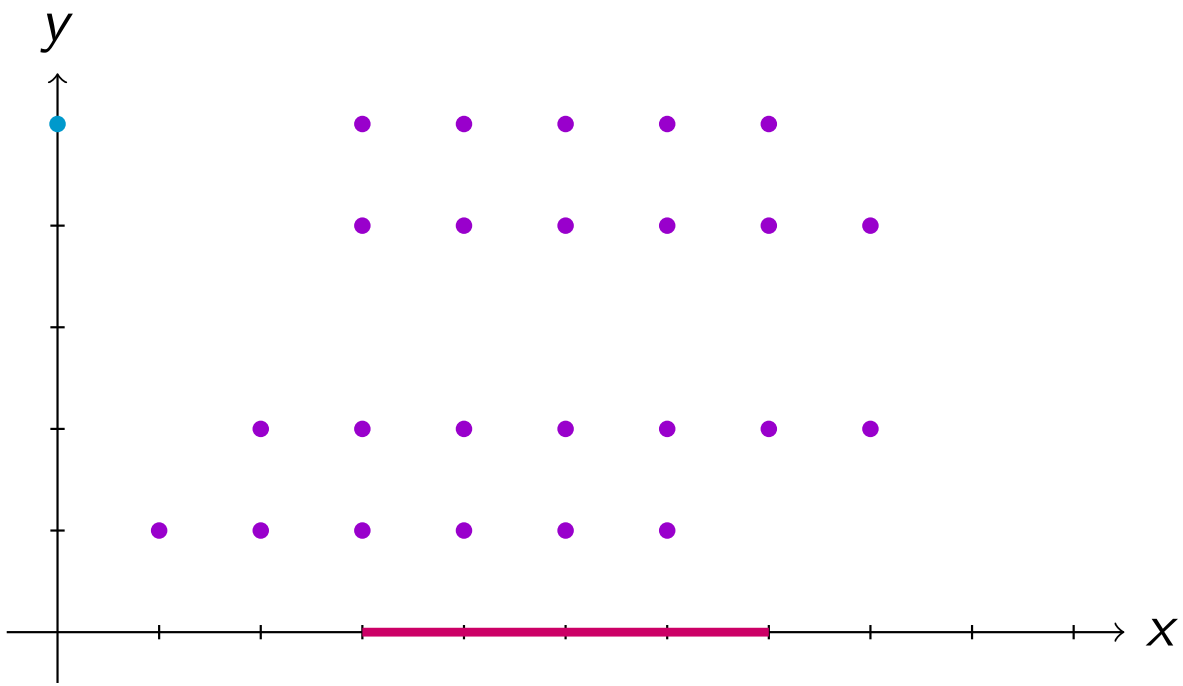

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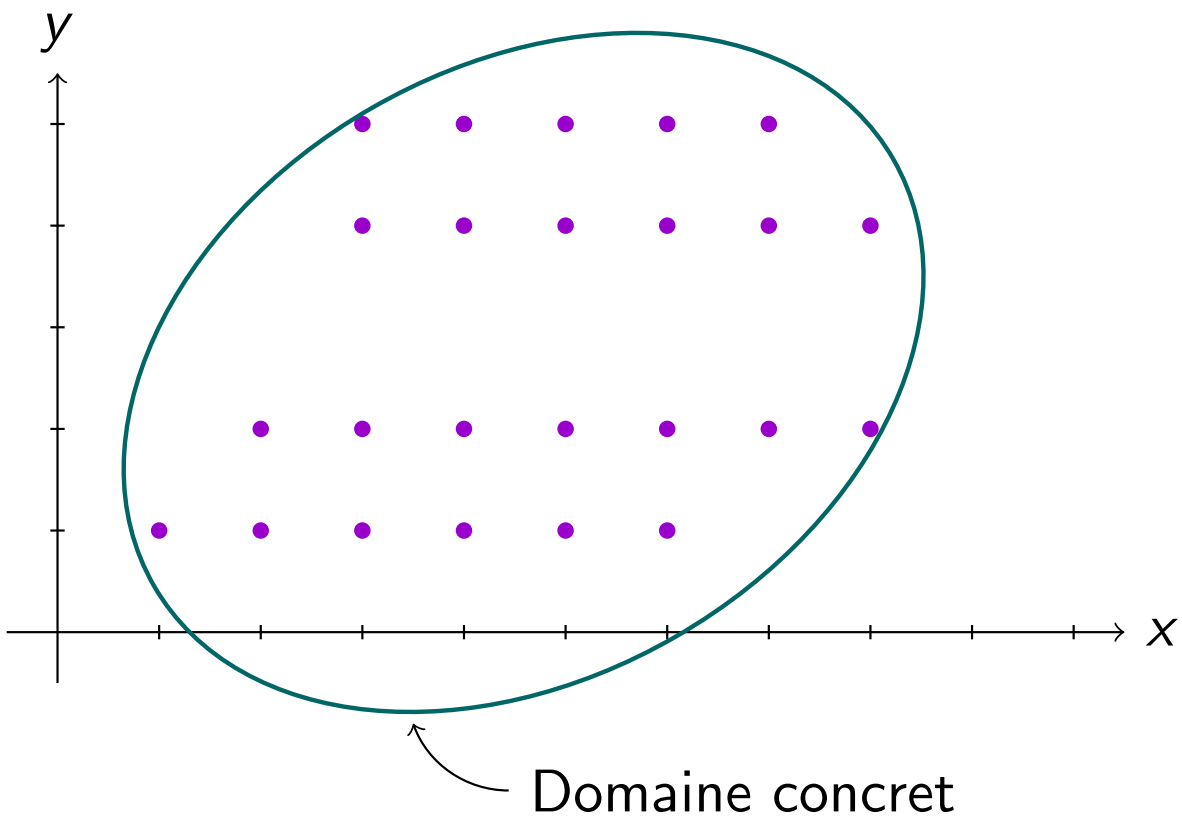

Remark

Calculer le domaine concret peut être indécidable ou trop coûteux

Schéma itératif

Itérations de Gauss-Seidel avec les intervalles

$X_1 =$
```
y ← 1
```
$X_2 =$
```
x ← random(1, 5)
```
$X_3 =$
$X_4' =$
```
while x≤8 and y<3 do
```
  $X_4 =$
```
  x ← x+y
```
  $X_5 =$
```
  y ← 2*y
```
  $X_6 =$
$X_7 =$
```
x ← x-1
```
$X_8 =$
```
y ← y+1
```
$X_9 =$

Schéma itératif

Itérations de Gauss-Seidel avec les intervalles

$X_1 = \top$
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while x≤8 and y<3 do
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X_2 &= \{\!|\texttt{y} \leftarrow 1|\!\}X_1 \\
X_3 &= \{\!|\texttt{x} \leftarrow \texttt{random(1, 5)}|\!\}X_2 \\
X_4' &= X_3 \cup X_6 \\
X_4 &= \{\!|\texttt{x}\leq 8 \text{ and } \texttt{y}<3|\!\}X_4' \\
X_5 &= \{\!|\texttt{x} \leftarrow \texttt{x+y}|\!\}X_4 \\
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X_8 &= \{\!|\texttt{x} \leftarrow \texttt{x-1}|\!\}X_7 \\
X_9 &= \{\!|\texttt{y} \leftarrow \texttt{y+1}|\!\}X_8
\end{aligned}
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Itérations de Gauss-Seidel avec les intervalles

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Schéma itératif

Itérations de Gauss-Seidel avec les intervalles

$X_1 = \{x \in \mathbb{Z},\ y \in \mathbb{Z}\}$
`y ← 1`
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$X'_4 = \{x \in [1,6],\ y = [1,2]\}$
`while x≤8 and y<3 do`
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  `x ← x+y`
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  $X_6 = \{x \in [2,8],\ y = [2,4]\}$
$X_7 = \bot$
`x ← x-1`
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$$
\begin{aligned}
X_1 &= \top \\
X_2 &= [\![\texttt{y ← 1}]\!] X_1 \\
X_3 &= [\![\texttt{x ← random(1, 5)}]\!] X_2 \\
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X_4 &= [\![\texttt{x≤8 and y<3}]\!] X'_4 \\
X_5 &= [\![\texttt{x ← x+y}]\!] X_4 \\
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$X_9 = \bot$

$$
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$$
\begin{aligned}
X_1 &= \top \\
X_2 &= \{\!|\texttt{y} \leftarrow 1|\!\}X_1 \\
X_3 &= \{\!|\texttt{x} \leftarrow \texttt{random(1, 5)}|\!\}X_2 \\
X'_4 &= X_3 \cup X_6 \\
X_4 &= \{\!|\texttt{x}\leq 8 \text{ and } \texttt{y}<3|\!\}X'_4 \\
X_5 &= \{\!|\texttt{x} \leftarrow \texttt{x+y}|\!\}X_4 \\
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```

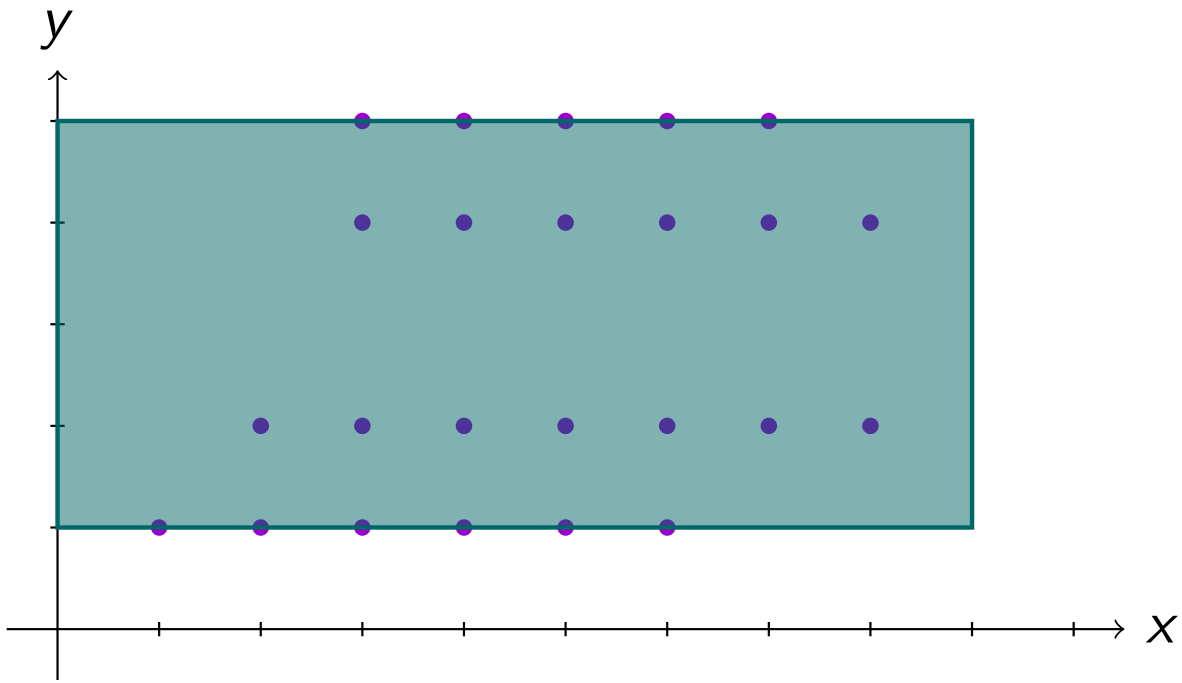

Cousot, P. and Cousot, R. (1976).
Static determination of dynamic properties of programs.
In *Proceedings of the 2nd International Symposium on Programming*, pages 106–130.