# Problem Solving
# Constraint Programming

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Notes



## Greedy Algorithm

### Principle

- At each step, a choice is made, the one that seems the best at that moment
- Builds a solution step by step
  - without revisiting previous decisions
  - by making at each step the choice that seems the best
  - hoping to achieve a global optimal result
- Greedy approach
  - depending on the problem, no guarantee of optimality (greedy heuristic)
  - low cost (compared to exhaustive enumeration)
  - intuitive choice



Notes

## Local Search

### Principle

- Start from an initial solution
- At each step, modify the solution
  - trying to improve the value of the objective function
  - hoping to achieve the global optimum
- Local approach
  - depending on the problem, no guarantee of optimality (heuristic)
  - low cost

### Remark

When there are no objective function *Constraint Based Local Search*



Notes



## Constraint Programming

- Tree Search
  - find a solution
  - find all solutions
  - find an optimal solution
  - prove the non-existence of a solution
- Complete Approach
  - guarantees optimality
  - more costly



Notes

## Send More Money

### Description

$$\begin{array}{r} S\,E\,N\,D \\ +\;\; M\,O\,R\,E \\ \hline M\,O\,N\,E\,Y \end{array}$$

### Contraintes possibles

$$\begin{array}{lrcrcrcrcr} & & & s{*}1000 & + & e{*}100 & + & n{*}10 & + & d \\ C_1 : & & + & m{*}1000 & + & o{*}100 & + & r{*}10 & + & e \\ & = m * 10000 & + & o*1000 & + & n{*}100 & + & e{*}10 & + & y \end{array}$$

| | | | | |
|---|---|---|---|---|
| $C_2 : s \neq e$ | $C_3 : s \neq n$ | $C_4 : s \neq d$ | $C_5 : s \neq m$ | $C_6 : s \neq o$ |
| $C_7 : s \neq r$ | $C_8 : s \neq y$ | $C_9 : e \neq n$ | $C_{10} : e \neq d$ | $C_{11} : e \neq m$ |
| $C_{12} : e \neq o$ | ... | $C_{27} : o \neq r$ | $C_{28} : o \neq y$ | $C_{29} : r \neq y$ |



Notes

Constraint Programming

## Send More Money

### Description

$$\begin{array}{r} r_4\,r_3\,r_2\,r_1\;\; \\ S\,E\,N\,D \\ +\;\; M\,O\,R\,E \\ \hline M\,O\,N\,E\,Y \end{array}$$

### Contraintes possibles

$$\begin{array}{lrcl} C_1 : & d + e = y + 10 * r_1 & & r_1 \in \{0,1\} \\ C_2 : & r_1 + n + r = e + 10 * r_2 & & r_2 \in \{0,1\} \\ C_3 : & r_2 + e + o = n + 10 * r_3 & & r_3 \in \{0,1\} \\ C_4 : & r_3 + s + m = o + 10 * r_4 & & r_4 \in \{0,1\} \\ C_5 : & r_4 = m & & \end{array}$$

| | | | | |
|---|---|---|---|---|
| $C_6 : s \neq e$ | $C_7 : s \neq n$ | $C_8 : s \neq d$ | $C_9 : s \neq m$ | $C_{10} : s \neq o$ |
| $C_{11} : s \neq r$ | $C_{12} : s \neq y$ | $C_{13} : e \neq n$ | $C_{14} : e \neq d$ | $C_{15} : e \neq m$ |
| $C_{16} : e \neq o$ | ... | $C_{31} : o \neq r$ | $C_{32} : o \neq y$ | $C_{33} : r \neq y$ |



Notes

## How to solve a CSP?



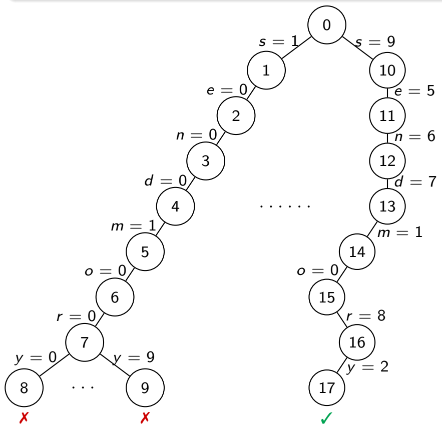
Notes



# Generate and Test

Naive method

Generate all possible assignments and check if they correspond to solutions




**Remark**

To find the only solution, generates:

- $9^2 * 10^6 = 81\ 000\ 000$ leaves with the first model
- $2^4 * 9^2 * 10^6 = 1\ 296\ 000\ 000$ with the second

**Can we do better?**



Notes

## Generate and Test

**Coloriage de carte**

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
- $D_1 = \cdots = D_7$
  $= \{\bullet, \bullet, \bullet\}$
- $C_1 : v_1 \neq v_2$
  $C_2 : v_1 \neq v_3$
  $C_3 : v_2 \neq v_3$
  $C_4 : v_2 \neq v_4$
  $C_5 : v_3 \neq v_4$
  $C_6 : v_3 \neq v_5$
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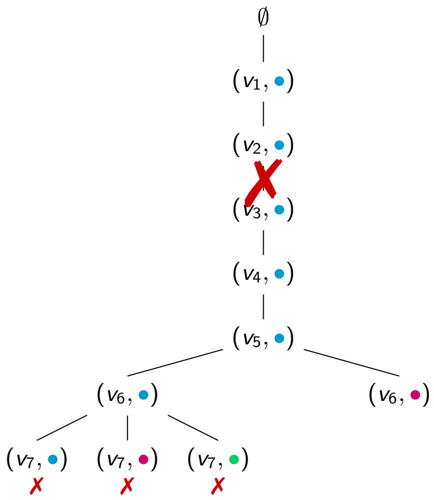




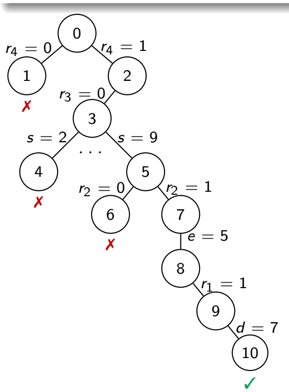

Notes



## Forward Checking

As soon as a variable is assigned, we try to filter the values for the other variables

- Replace the variable with its value in all constraints
- Filtering can be applied if a constraint has only one remaining variable




- $\{C_4 : r_3 + s + m = o + 10 * r_4,$
  $C_5 : r_4 = m\}$
  - $r_4 = 0$
    $\Rightarrow r_3 + s + m = o \wedge m = 0$ ✗
  - $r_4 = 1$
    $\Rightarrow r_3 + s + m = o + 10 \wedge m = 1$
    $\Rightarrow r_3 + s + 1 = o + 10$

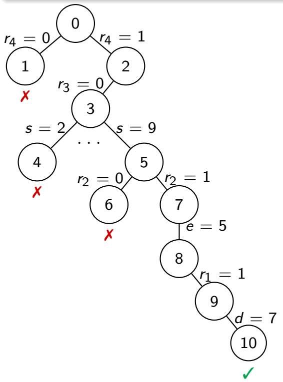


Notes

# Forward Checking

As soon as a variable is assigned, we try to filter the values for the other variables

- Replace the variable with its value in all constraints
- Filtering can be applied if a constraint has only one remaining variable




**Remark**

To find the only solution, generates:

- 483 840 leaves with the first model
- 57 with the second

**Why wait for an assignment?**



Notes



# Method with Filtering

The **2 key steps** of constraint programming!

**Propagation**

Removes inconsistent values from the domains, meaning values that cannot be part of a solution.

**Exploration**

Assigns a value to a variable.



Notes

## Propagation

### Consistency for a constraint

Different types of consistency:

- Generalized arc consistency [Mackworth, 1977b]
- Path consistency [Montanari, 1974]
- Bound consistency [van Hentenryck et al., 1995]
- ...

All of these rely on the notion of support



Notes



## Propagation

### Consistency for a constraint

Definition (Support)

Let $v_1, \ldots, v_n$ be variables with finite discrete domains $D_1, \ldots, D_n$, and $C$ be a constraint. The value $x_i \in D_i$ has a **support** if and only if $\forall j \in [1, n], j \neq i, \exists x_j \in D_j$ such that $C(x_1, \ldots, x_n)$ is true

Example

$C : r_4 = m$ with $D_{r_4} = [0, 1]$ and $D_m = [1, 9]$

- 1 for $r_4$ has a support: 1 for $m$ because $C(1, 1)$ is true
- 0 for $r_4$ does not have a support: $\forall x_m \in D_m, C(0, x_m)$ is false



Notes

## Propagation

### Consistency for a constraint

**Definition (Support)**

Let $v_1, \dots, v_n$ be variables with finite discrete domains $D_1, \dots, D_n$, and $C$ be a constraint. The value $x_i \in D_i$ has a **support** if and only if $\forall j \in [1, n], j \neq i, \exists x_j \in D_j$ such that $C(x_1, \dots, x_n)$ is true

**Example**

$C : v_1 \neq v_2$ with $D_1 = D_2 = \{\textcolor{cyan}{\bullet}, \textcolor{magenta}{\bullet}, \textcolor{green}{\bullet}\}$

- $\textcolor{cyan}{\bullet}$ for $v_1$ has a support: $\textcolor{magenta}{\bullet}$ for $v_2$ because $C(\textcolor{cyan}{\bullet}, \textcolor{magenta}{\bullet})$ is true
- $\textcolor{magenta}{\bullet}$ for $v_1$ has a support: $\textcolor{cyan}{\bullet}$ for $v_2$ because $C(\textcolor{magenta}{\bullet}, \textcolor{cyan}{\bullet})$ is true
- $\textcolor{green}{\bullet}$ for $v_1$ has a support: $\textcolor{cyan}{\bullet}$ for $v_2$ because $C(\textcolor{green}{\bullet}, \textcolor{cyan}{\bullet})$ is true



Notes



## Consistencies

**Definition (Bound consistency)**

Let $v_1, \dots, v_n$ be variables with finite discrete domains $D_1, \dots, D_n$, and $C$ be a constraint. The domains are said to be **bound-consistent** (BC) for $C$ if and only if $\forall i \in [1, n], D_i = [a_i, b_i]$, where $a_i$ and $b_i$ have a support.

**Example**

Consider two variables $v_1, v_2$ with domains $D_1 = D_2 = [-1, 4]$ and the constraint $v_1 = 2v_2$. The bound-consistent domains for this constraint are $D_1 = [0, 4]$ and $D_2 = [0, 2]$



Notes

## Consistencies

### Definition (Generalized Arc Consistency)

Let $v_1, \dots, v_n$ be variables with finite discrete domains $D_1, \dots, D_n$, and $C$ be a constraint. The domains are said to be **generalized arc-consistent** (GAC) for $C$ if and only if $\forall i \in [1, n], \forall x \in D_i, x$ has a support.

### Example

Let $v_1, v_2$ be two variables with domains $D_1 = D_2 = [-1, 4]$ and the constraint $v_1 = 2v_2$. The arc-consistent domains for this constraint are $D_1 = \{0, 2, 4\}$ and $D_2 = \{0, 1, 2\}$



Notes



## Arc Consistency

### Several implementations

- AC1 and AC3 [Mackworth, 1977a]
- AC4 [Mohr and Henderson, 1986]
- AC5 [van Hentenryck et al., 1992]
- AC6 [Bessière, 1994]
- AC7 [Bessière et al., 1999]
- AC2001 [Bessière and Régin, 2001]
- AC3.2 and AC3.3 [Lecoutre et al., 2003]

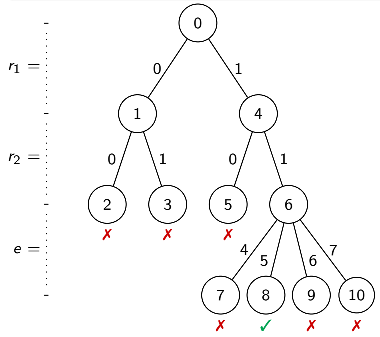


Notes

# Maintaining Generalized Arc Consistency

Two phases alternate:

- Propagation, using generalized arc consistency
- Exploration, making a choice



**Remark**

To find the only solution, generates:

- 6 leaves with the first model
- 7 with the second

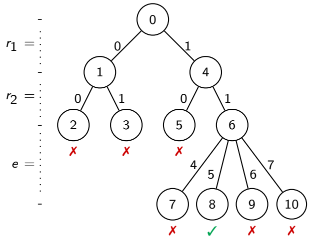


Notes

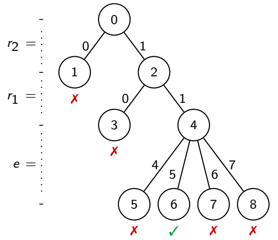


# Search Strategy

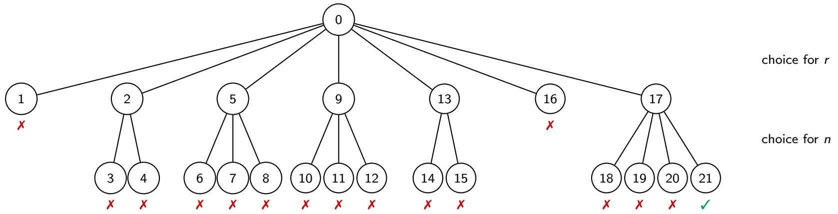
## The order of the variables matters



7 leaves



6 leaves



16 leaves



Notes

## Search Strategy

Choose a variable

- Having the smallest domain $(\mathrm{dom})$, First-fail [Haralick and Elliott, 1979]
  *"To succeed, try first where you are most likely to fail"*:
- Appearing in the greatest number of constraints $(\mathrm{deg})$
- $\mathrm{dom} + \mathrm{deg}$ [Brélaz, 1979]
- $\mathrm{dom}/\mathrm{deg}$ [Bessière and Régin, 1996]
- $\mathrm{dom}/w\mathrm{deg}$ [Boussemart et al., 2004]
- ...



Notes



## Search Strategy

Coloriage de carte

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
- $D_1 = \cdots = D_7 = \{\bullet, \bullet, \bullet\}$
- $C_1 : v_1 \neq v_2$
  $C_2 : v_1 \neq v_4$
  $C_3 : v_2 \neq v_3$
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  $C_5 : v_3 \neq v_4$
  $C_6 : v_3 \neq v_5$
  $C_7 : v_4 \neq v_5$
  $C_8 : v_4 \neq v_6$
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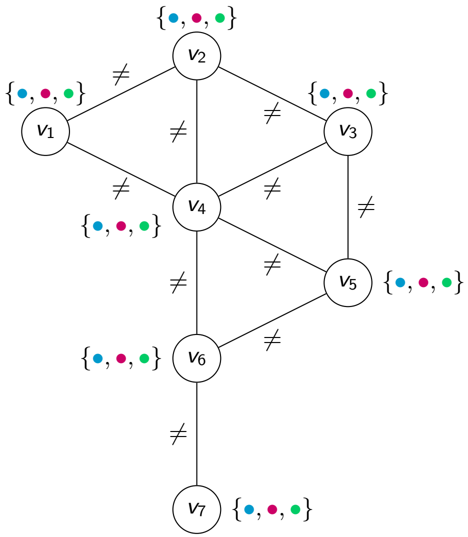
  $C_{10} : v_6 \neq v_7$






Notes

## Search Strategy - dom

### Coloriage de carte

- $\mathcal{V} = \{v_1, \dots, v_7\}$
- $D_1 = \dots = D_7 = \{\bullet, \bullet, \bullet\}$
- $C_1 : v_1 \neq v_2$
  $C_2 : v_1 \neq v_4$
  $C_3 : v_2 \neq v_3$
  $C_4 : v_2 \neq v_4$
  $C_5 : v_3 \neq v_4$
  $C_6 : v_3 \neq v_5$
  $C_7 : v_4 \neq v_5$
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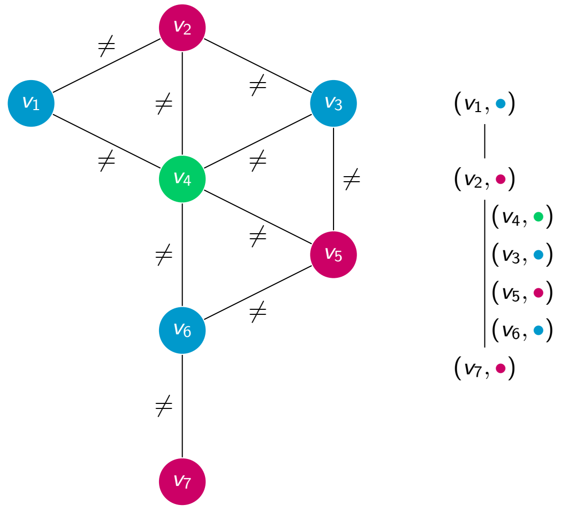
  $C_9 : v_5 \neq v_6$
  $C_{10} : v_6 \neq v_7$






Notes



## Search Strategy - deg

### Coloriage de carte

- $\mathcal{V} = \{v_1, \dots, v_7\}$
- $D_1 = \dots = D_7 = \{\bullet, \bullet, \bullet\}$
- $C_1 : v_1 \neq v_2$
  $C_2 : v_1 \neq v_4$
  $C_3 : v_2 \neq v_3$
  $C_4 : v_2 \neq v_4$
  $C_5 : v_3 \neq v_4$
  $C_6 : v_3 \neq v_5$
  $C_7 : v_4 \neq v_5$
  $C_8 : v_4 \neq v_6$
  $C_9 : v_5 \neq v_6$
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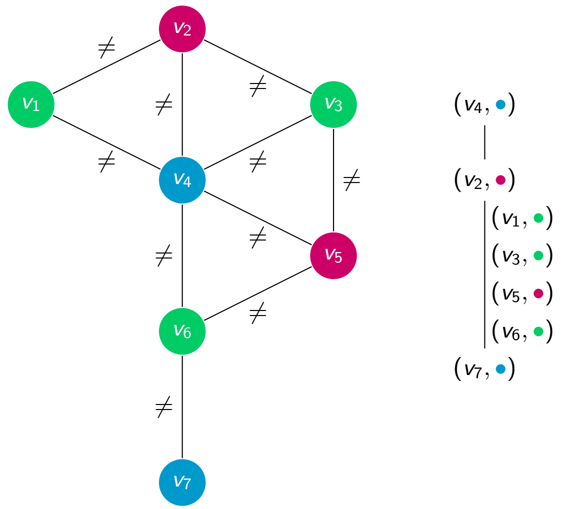




Notes

## Does it work all the time?



Notes



# Limites

**Coloriage de carte**

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
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- $C_1 : v_1 \neq v_2$
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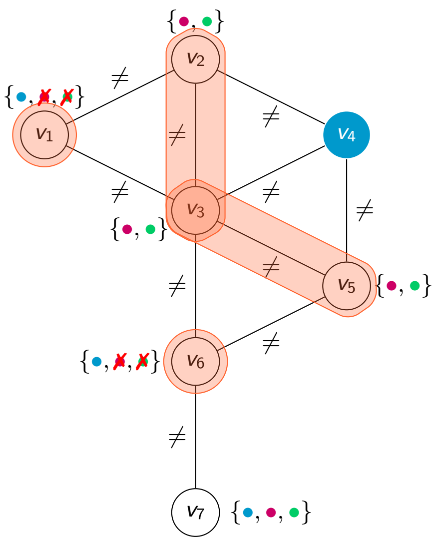
  $C_9 : v_5 \neq v_6$
  $C_{10} : v_6 \neq v_7$






Notes

## Limites

### Coloriage de carte

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
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  $C_4 : v_2 \neq v_4$
  $C_5 : v_3 \neq v_4$
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  $C_8 : v_4 \neq v_5$
  $C_9 : v_5 \neq v_6$
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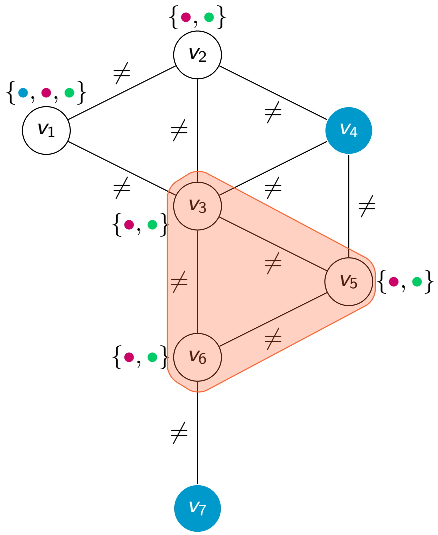




Notes



## Global Constraints

Allows representing a set of constraints

- Facilitates modeling
- Dedicated algorithm to remove inconsistent values from domains

### Constraint Catalog [Beldiceanu et al., 2010]

The most well-known

- alldifferent
- cycle
- global_cardinality
- nvalue
- element



Notes

## alldifferent Constraint

First presented in [Lauriere, 1978]
Returns **true** if all variables are pairwise different

**Example**

The difference constraints in the send + more = money problem can be rewritten as
$\mathrm{alldifferent}(s, e, n, d, m, o, r, y)$



Notes



## alldifferent Constraint
### Map Coloring

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
- $D_1 = \cdots = D_7 = \{\bullet, \bullet, \bullet\}$
- $C_1 : v_1 \neq v_2$
  $C_2 : v_1 \neq v_3$
  $C_3 : v_2 \neq v_3$
  $C_4 : v_2 \neq v_4$
  $C_5 : v_3 \neq v_4$
  $C_6 : v_3 \neq v_5$
  $C_7 : v_3 \neq v_6$
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  $C_9 : v_5 \neq v_6$
  $C_{10} : v_6 \neq v_7$

- $\mathcal{V} = \{v_1, \ldots, v_7\}$
- $D_1 = \cdots = D_7 = \{\bullet, \bullet, \bullet\}$
- $C_1 : \mathrm{alldifferent}(v_1, v_2, v_3)$
  $C_2 : \mathrm{alldifferent}(v_2, v_3, v_4)$
  $C_3 : \mathrm{alldifferent}(v_3, v_4, v_5)$
  $C_4 : \mathrm{alldifferent}(v_3, v_5, v_6)$
  $C_5 : v_6 \neq v_7$



Notes

## alldifferent Constraint

- Not just syntactic sugar
  - Arc-consistency
    - Developed independently by [Costa, 1994] and [Régin, 1994]
    - Based on graph theory
  - Bound-consistency
    - Developed by [Puget, 1998] and later improved by [Mehlhorn and Thiel, 2000] and [Lopez-Ortiz et al., 2003]
    - Based on the concept of Hall's interval



Notes



## Value Graph

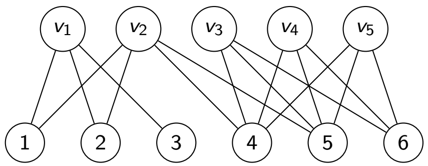
**Definition (Value Graph)**

From the variables and domains of a CSP, we can create a bipartite graph, called the **value graph**

- The vertices correspond to the variables and the values
- An edge connects a variable $v_i$ and a value $x$ if $x \in D_i$

**Example**

- $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$
- $D_1 = \{1, 2, 3\}$, $D_2 = \{1, 2, 4, 5\}$, et $D_3 = D_4 = D_5 = \{4, 5, 6\}$



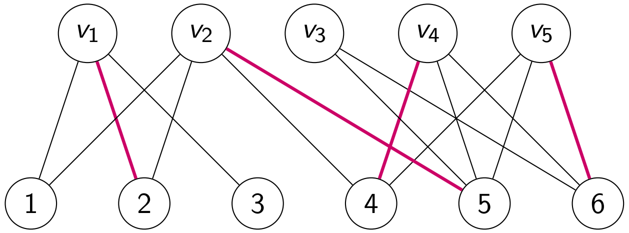



Notes

## Graph theory

### Definition (Matching)

Given a graph $G = (V, E)$, a subset $M$ of the edges $E$ is called a **matching** if and only if no two edges share a vertex,





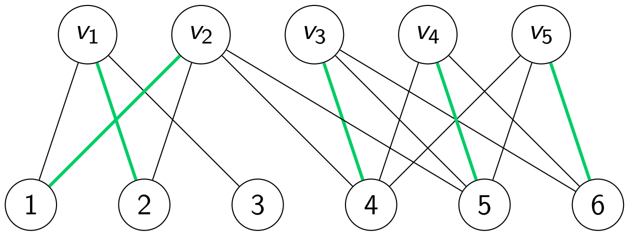

Notes



## Graph theory

### Definition (Maximal Matching)

A matching is said to be **maximal** if it contains the maximum number of edges possible.

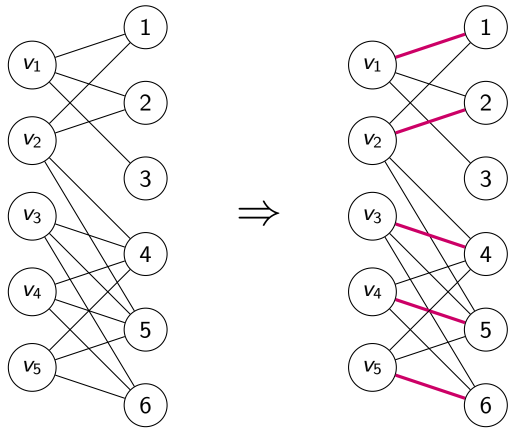



The Hopcroft-Karp algorithm [Hopcroft and Karp, 1973] allows for calculating the maximal matching in a bipartite graph



Notes

## Hopcroft-Karp algorithm






Notes



## Strongly Connected Component

**Definition (Directed Graph)**

A **directed** graph $G = (V, E)$ is a graph where the edges have a direction, and they are called **arcs**

**Definition (Strongly Connected Component)**

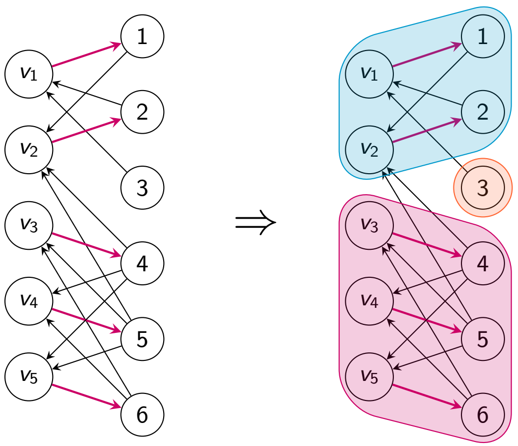
Given a **directed** graph $G = (V, E)$, a **strongly connected component** is a maximal set of vertices such that for each vertex in the set, there exists a path to every other vertex in the set

Tarjan's algorithm [Tarjan, 1972] efficiently computes the strongly connected components in a graph



Notes

## Tarjan algorithm






Notes

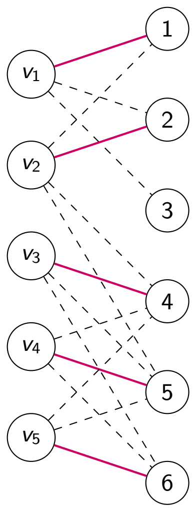


## alldifferent: propagation for arc-consistency

**Exemple**

- $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$
- $D_1 = \{1, 2, 3\}$, $D_2 = \{1, 2, 4, 5\}$, et $D_3 = D_4 = D_5 = \{4, 5, 6\}$

- We find a maximal matching $\Rightarrow$ **a solution**



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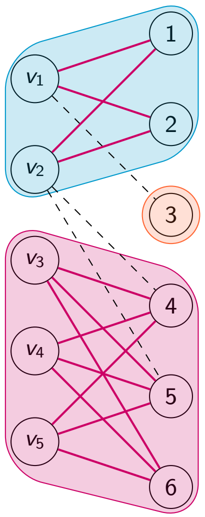
- We find a maximal matching $\Rightarrow$ **a solution**
- We search for strongly connected components $\Rightarrow$ **permutations**






Notes



## alldifferent: propagation for arc-consistency

Exemple

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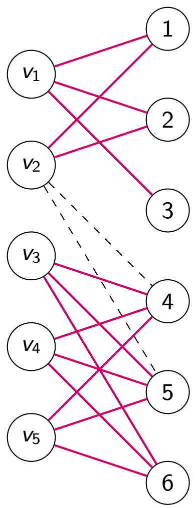
- We find a maximal matching $\Rightarrow$ **a solution**
- We search for strongly connected components $\Rightarrow$ **permutations**
- We add the isolated values to the initial domains





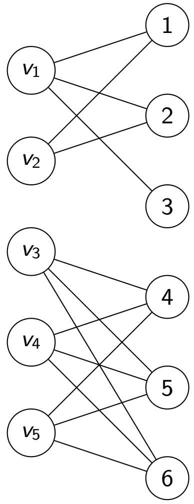

Notes

## alldifferent: propagation for arc-consistency

**Exemple**

- $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$
- $D_1 = \{1, 2, 3\}$, $D_2 = \{1, 2\}$, et $D_3 = D_4 = D_5 = \{4, 5, 6\}$

- We find a maximal matching $\Rightarrow$ **a solution**
- We search for strongly connected components $\Rightarrow$ **permutations**
- We add the isolated values to the initial domains






Notes



## Hall's Interval

**Definition**

Let $(v_1, \dots, v_n)$ be variables with finite discrete domains $(D_1, \dots, D_n)$. Given an interval $I$, we define $K_I = \{v_i \mid D_i \subseteq I\}$. $I$ is a **Hall's interval** if $|I| = |K_I|$.

**Example**

Consider the following problem:

- $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$
- $D_1 = [1, 3]$, $D_2 = [1, 5]$, et $D_3 = D_4 = D_5 = [4, 6]$
- $I = [4, 6]$ is a Hall's interval because $K_I = \{v_3, v_4, v_5\}$ and we have $|I| = |K_I|$
- $I = [1, 3]$ is not a Hall's interval because $K_I = \{v_1\}$ and $|I| \neq |K_I|$



Notes

## alldifferent: propagation for bound-consistency

- For each lower bound $a$ and upper bound $b$ of the domains, we check if $I = [a, b]$ is a Hall's interval
- If $I$ is a Hall's interval, we can remove the values in $I$ from the domains of variables in $\mathcal{V} \setminus K_I$

Exemple

Consider the following problem:

- $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5\}$
- $D_1 = [1, 3]$, $D_2 = [1, 3]$, et $D_3 = D_4 = D_5 = [4, 6]$
- $I = [1, 6]$ is not a Hall's interval
- $I = [1, 5]$ is not a Hall's interval
- $I = [1, 3]$ is not a Hall's interval
- $I = [4, 5]$ is not a Hall's interval
- $I = [4, 6]$ is a Hall's interval $\Rightarrow$ **we remove the values 4, 5, 6 from the domains of variables not in $K_I$**



Notes



## global_cardinality Constraint

First presented in [Oplobedu et al., 1989]

$$\text{global\_cardinality}(\underbrace{\{v_1, \ldots, v_n\}}_{\text{Variables}}, \underbrace{\{x_1, \ldots, p\}}_{\text{Values}}, \underbrace{\{nb_1, \ldots, nb_p\}}_{\text{Occurrences}})$$

Returns **true** if among the variables $\{v_1, \ldots, v_n\}$, there are $nb_i$ variables having the value $x_i$

Exemple

$\text{global\_cardinality}(\{v_1, v_2, v_3, v_4, v_5, v_6\}, \{0, 1\}, \{2, 4\})$

In some cases, we can express an alldifferent using a global_cardinality

$\text{alldifferent}(v_1, v_2, v_3) = \text{global\_cardinality}(\{v_1, v_2, v_3\}, \{\bullet, \bullet, \bullet\}, \{1, 1, 1\})$



Notes

## global_cardinality Constraint

- Arc-consistency
  - Developed by [Régin, 1996]
  - Based on a flow algorithm
- Bound-consistency
  - Developed by [Quimper et al., 2003]
  - Based on the concept of Hall's interval
  - Developed by [Katriel and Thiel, 2003]
  - Based on convexity to improve the efficiency of the flow algorithm



Notes



## Sports Schedule

### Description

- $n$ teams, $n-1$ weeks, and $n/2$ periods
- each pair of teams plays exactly once
- each team plays one match every week
- each team plays at most 2 times in the period

### Example (Possible solution)

| | S1 | S2 | S 3 | S4 | S5 | S6 | S7 |
|---|---|---|---|---|---|---|---|
| P1 | 1 vs 2 | 1 vs 3 | 5 vs 8 | 4 vs 7 | 4 vs 8 | 2 vs 6 | 3 vs 5 |
| P2 | 3 vs 4 | 2 vs 8 | 1 vs 4 | 6 vs 8 | 2 vs 5 | 1 vs 7 | 6 vs 7 |
| P3 | 5 vs 6 | 4 vs 6 | 2 vs 7 | 1 vs 5 | 3 vs 7 | 3 vs 8 | 1 vs 8 |
| P4 | 7 vs 8 | 5 vs 7 | 3 vs 6 | 2 vs 3 | 1 vs 6 | 4 vs 5 | 2 vs 4 |



Notes

# Magic Sequence

### Description

A magic sequence of length $n$ is a sequence of integers $v_0, \ldots, v_{n-1}$, where each integer $i \in \{0, \ldots, n-1\}$ appears exactly $v_i$ times in the sequence

### Magic Sequence ($n = 10$)

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $v_i$ | 6 | 2 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |



Notes



# Langford Sequence

### Description

A Langford sequence is a sequence of integers $v_1, \ldots, v_{k \times n}$, where each integer $i \in \{1, \ldots, n\}$ appears exactly $k$ times, and the two successive occurrences of $i$ are separated by a distance of $i$

We consider here only the case for $k = 2$

### Langford Sequence ($n = 7$)

4

7 3 6 2 5 3 2 4 7 6 5 1 4 1

7 1



Notes

# Alice and Bob are Going to Work

## Description

- Alice goes to work by car (30 to 40 minutes) or by bus (at least 60 min)
- Bob goes by bike (40 or 50 min) or by motorbike (20 to 30 min)
- This morning:
  - Alice left her house between 7:10 AM and 7:20 AM
  - Bob arrived at work between 8:00 AM and 8:10 AM
  - Alice arrived 10 to 20 minutes after Bob left

1. Model this problem
2. Is the story consistent?
3. When did Bob leave? Is it possible that he took his bike?
4. Is the story consistent if we add that:
   - Alice's car is broken down
   - Alice and Bob met on the way



Notes



# Binairo – 2018 Exam

## Description

A Belgian game, based on a square grid with only the digits 0 and 1. On each row and each column:

- there are as many 0's as 1's
- there cannot be more than 2 identical digits next to each other

No two rows or columns can be identical.

## Example Grid

| | | | | | |
|---|---|---|---|---|---|
| | 0 | | | 1 | 0 |
| | 1 | 0 | | | 1 |
| | | | | | |
| 1 | 0 | | 0 | 1 | |
| 1 | | | | | |
| | | | | 1 | 1 |



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## Notes