Problem Solving
Modelization

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Formulation

Model
- $\mathcal{V} = \{v_1, \ldots, v_n\}$: variables
- $\mathcal{D} = \{D_1, \ldots, D_n\}$: domaines
- $\mathcal{C} = \{C_1, \ldots, C_p\}$: constraints

Notes

Notes

Coloriage de carte

Description

- 3 colors: blue, pink, et green
- 2 bordering regions have different colors

- What are the unknowns?
  The colors of the regions. We have 7 variables:
  $\mathcal{V} = \{v_1, v_2, \ldots, v_7\}$ What are the possible values? The colors. We have $D_1 = \cdots = D_7 = \{\bullet, \bullet, \bullet\}$

Coloriage de carte

Constraints

$$
\begin{array}{llll}
C_1 & : & v_1 \neq v_2 \qquad & C_2 & : & v_1 \neq v_4 \\
C_3 & : & v_2 \neq v_3 & C_4 & : & v_2 \neq v_4 \\
C_5 & : & v_3 \neq v_4 & C_6 & : & v_3 \neq v_5 \\
C_7 & : & v_4 \neq v_5 & C_8 & : & v_4 \neq v_6 \\
C_9 & : & v_5 \neq v_6 & C_{10} & : & v_6 \neq v_7
\end{array}
$$

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Description

$$\begin{array}{r} S\,E\,N\,D \\ +\ \ M\,O\,R\,E \\ \hline M\,O\,N\,E\,Y \end{array}$$

Each letter represents a different number between 0 and 9. We want to know the value of each letter, knowing that the first letter of each word cannot be equal to 0

- What are the unknowns? The letters. We therefore have 8 variables $\mathcal{V} = \{s, e, n, d, m, o, r, y\}$ What are the possible values? Between 0 and 9, except for s and m. We have $D_s = D_m = [1, 9], D_e = D_n = D_d = D_o = D_r = D_y = [0, 9]$

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Description

$$\begin{array}{r} S\,E\,N\,D \\ +\ \ M\,O\,R\,E \\ \hline M\,O\,N\,E\,Y \end{array}$$

Possible constraints

C_1 :
$$\begin{aligned} & s*1000 \ + \ e*100 \ + \ n*10 \ + \ d \\ & + \ m*1000 \ + \ o*100 \ + \ r*10 \ + \ e \\ = \ & m*10000 \ + \ o*1000 \ + \ n*100 \ + \ e*10 \ + \ y \end{aligned}$$

$C_2\ :\ s \neq e$	$C_3:\ s \neq n$	$C_4\ :\ s \neq d$	$C_5\ :\ s \neq m$	$C_6\ :\ s \neq o$
$C_7\ :\ s \neq r$	$C_8:\ s \neq y$	$C_9\ :\ e \neq n$	$C_{10}:\ e \neq d$	$C_{11}:\ e \neq m$
$C_{12}:\ e \neq o$	...	$C_{27}:\ o \neq r$	$C_{28}:\ o \neq y$	$C_{29}:\ r \neq y$

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Description

$$r_4\, r_3\, r_2\, r_1$$
$$\mathsf{S\ E\ N\ D}$$
$$+\ \mathsf{M\ O\ R\ E}$$
$$\overline{\mathsf{M\ O\ N\ E\ Y}}$$

Possible constraints

C_1 : $\qquad d + e = y + 10 * r_1 \qquad r_1 \in \{0,1\}$

C_2 : $\quad r_1 + n + r = e + 10 * r_2 \qquad r_2 \in \{0,1\}$

C_3 : $\quad r_2 + e + o = n + 10 * r_3 \qquad r_3 \in \{0,1\}$

C_4 : $\quad r_3 + s + m = o + 10 * r_4 \qquad r_4 \in \{0,1\}$

C_5 : $\qquad\qquad r_4 = m$

$C_6\ :\ s \neq e$	$C_7\ :\ s \neq n$	$C_8\ :\ s \neq d$	$C_9\ :\ s \neq m$	$C_{10}\ :\ s \neq o$
$C_{11}\ :\ s \neq r$	$C_{12}\ :\ s \neq y$	$C_{13}\ :\ e \neq n$	$C_{14}\ :\ e \neq d$	$C_{15}\ :\ e \neq m$
$C_{16}\ :\ e \neq o$	...	$C_{31}\ :\ o \neq r$	$C_{32}\ :\ o \neq y$	$C_{33}\ :\ r \neq y$

Zebra

Description

Five consecutive houses

- **Different colors**
  blue, yellow, orange, red, green

- **Inhabited by men of different nationalities**
  English, Spanish, Japanese, Norwegian, Ukrainian

- **Each one has a different pet**
  dog, horse, snail, fox, zebra

- **Everyone has a different favorite drink**
  coffee, water, milk, tea, wine

- **Everyone smokes different brand of cigarettes**
  chesterfields, cravens, gitanes, kools, old golds

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Zebra

Description

1. The Norwegian lives in the first house
2. The house next to the Norwegian's is blue
3. The inhabitant of the third house drinks milk
4. The Englishman lives in the red house
5. The inhabitant of the green house drinks coffee
6. The inhabitant of the yellow house smokes kools
7. The orange house is right after the green one
8. The Spaniard has a dog
9. Ukrainian drinks tea
10. The Japanese smokes cravens
11. The old golds smoker has a snail
12. The gitanes smoker drinks wine
13. The chesterfields smoker's neighbor has a fox
14. The kools smoker's neighbor has a horse
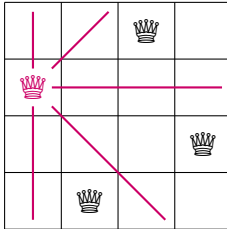
Who drinks water?
Who owns the zebra?

Zebra

- $\mathcal{V} = \{ m_{blue}, m_{yellow}, m_{orange}, m_{red}, m_{green}, m_{English}, m_{Spanish}, m_{Japanese},$ $m_{Norwegian}, m_{Ukrainian}, m_{dog}, m_{horse}, m_{snail}, m_{fox}, m_{zebra}, m_{coffee}, m_{water},$ $m_{milk}, m_{tea}, m_{wine}, m_{chesterfields}, m_{cravens}, m_{gitanes}, m_{kools}, m_{oldgolds} \}$

- $\forall v \in \mathcal{V},\ D_v = \{1, 2, 3, 4, 5\}$

N-queens

Description

- On a $n \times n$ chessboard
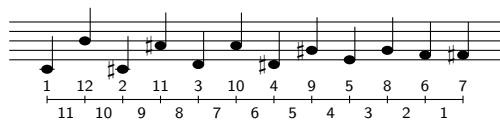- Place n queens so that no queen can capture another one

Music: all-interval series

Description

- In the 1920s, Arnold Schönberg created a compositional principle: dodecaphony
- Consider the chromatic scale, and look for a motif in which notes appear exactly once 0ntervals (between 2 successive notes) must be different

Example (Trivial solution)

Notes

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Magic Square

Description

- Place all the numbers from 1 to n^2 on an $n \times n$ square
- The sum of each row, each column, and both diagonals must be equal

17	24	1	8	15	→ 65
23	5	7	14	16	→ 65
4	6	13	20	22	→ 65
10	12	19	21	3	→ 65
11	18	25	2	9	→ 65

$\swarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$ $\searrow$

65   65 65 65 65 65   65

Latin square

Description

Given n colors, a Latin square is an $n \times n$ colored square such that:

- all cells are colored,
- each color appears exactly once in each row,
- each color appears exactly once in each column

Example (Solution for $n = 4$)

2	3	4	1
4	1	2	3
3	4	1	2
1	2	3	4

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