Problem Solving
Modelization

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Formulation

Model

- $\mathcal{V} = \{v_1, \ldots, v_n\}$: variables
- $\mathcal{D} = \{D_1, \ldots, D_n\}$: domaines
- $\mathcal{C} = \{C_1, \ldots, C_p\}$: constraints

Coloriage de carte

Description

- 3 colors: blue, pink, et green
- 2 bordering regions have different colors

- What are the unknowns?
  The colors of the regions. We have 7 variables:
  $\mathcal{V} = \{v_1, v_2, \ldots, v_7\}$ What are the possible values?The colors. We have $D_1 = \cdots = D_7 = \{●, ●, ●\}$

Coloriage de carte

Constraints

$C_1 : v_1 \neq v_2$ $C_2 : v_1 \neq v_4$
$C_3 : v_2 \neq v_3$ $C_4 : v_2 \neq v_4$
$C_5 : v_3 \neq v_4$ $C_6 : v_3 \neq v_5$
$C_7 : v_4 \neq v_5$ $C_8 : v_4 \neq v_6$
$C_9 : v_5 \neq v_6$ $C_{10} : v_6 \neq v_7$

Send More Money

Description

$$\begin{array}{r} \mathrm{S\,E\,N\,D} \\ +\ \ \mathrm{M\,O\,R\,E} \\ \hline \mathrm{M\,O\,N\,E\,Y} \end{array}$$

Each letter represents a different number between 0 and 9. We want to know the value of each letter, knowing that the first letter of each word cannot be equal to 0

- What are the unknowns? The letters. We therefore have 8 variables $\mathcal{V} = \{s, e, n, d, m, o, r, y\}$ What are the possible values? Between 0 and 9, except for s and m. We have $D_s = D_m = [1, 9], D_e = D_n = D_d = D_o = D_r = D_y = [0, 9]$

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Description

$$\begin{array}{r} \mathrm{S\,E\,N\,D} \\ +\ \ \mathrm{M\,O\,R\,E} \\ \hline \mathrm{M\,O\,N\,E\,Y} \end{array}$$

Possible constraints

$$\begin{array}{llrcrcrcrcl} & & & & s{*}1000 & + & e{*}100 & + & n{*}10 & + & d \\ C_1 : & & & + & m{*}1000 & + & o{*}100 & + & r{*}10 & + & e \\ & = & m * 10000 & + & o*1000 & + & n{*}100 & + & e{*}10 & + & y \end{array}$$

$$\begin{array}{lllll} C_2 : s \neq e & C_3 : s \neq n & C_4 : s \neq d & C_5 : s \neq m & C_6 : s \neq o \\ C_7 : s \neq r & C_8 : s \neq y & C_9 : e \neq n & C_{10} : e \neq d & C_{11} : e \neq m \\ C_{12} : e \neq o & \ldots & C_{27} : o \neq r & C_{28} : o \neq y & C_{29} : r \neq y \end{array}$$

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Description

$$\begin{array}{rrrrrr}
 & r_4 & r_3 & r_2 & r_1 & \\
 & & S & E & N & D \\
+ & & M & O & R & E \\
\hline
 & M & O & N & E & Y
\end{array}$$

Possible constraints

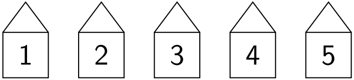
$$\begin{array}{lrcl}
C_1 : & d + e = y + 10 * r_1 & \quad & r_1 \in \{0, 1\} \\
C_2 : & r_1 + n + r = e + 10 * r_2 & & r_2 \in \{0, 1\} \\
C_3 : & r_2 + e + o = n + 10 * r_3 & & r_3 \in \{0, 1\} \\
C_4 : & r_3 + s + m = o + 10 * r_4 & & r_4 \in \{0, 1\} \\
C_5 : & r_4 = m & &
\end{array}$$

$$\begin{array}{lllll}
C_6 : s \neq e & C_7 : s \neq n & C_8 : s \neq d & C_9 : s \neq m & C_{10} : s \neq o \\
C_{11} : s \neq r & C_{12} : s \neq y & C_{13} : e \neq n & C_{14} : e \neq d & C_{15} : e \neq m \\
C_{16} : e \neq o & \ldots & C_{31} : o \neq r & C_{32} : o \neq y & C_{33} : r \neq y
\end{array}$$

Zebra

Description

Five consecutive houses



- Different colors
  blue, yellow, orange, red, green
- Inhabited by men of different nationalities
  English, Spanish, Japanese, Norwegian, Ukrainian
- Each one has a different pet
  dog, horse, snail, fox, zebra
- Everyone has a different favorite drink
  coffee, water, milk, tea, wine
- Everyone smokes different brand of cigarettes
  chesterfields, cravens, gitanes, kools, old golds

Zebra

Description

1. The Norwegian lives in the first house
2. The house next to the Norwegian's is blue
3. The inhabitant of the third house drinks milk
4. The Englishman lives in the red house
5. The inhabitant of the green house drinks coffee
6. The inhabitant of the yellow house smokes kools
7. The orange house is right after the green one
8. The Spaniard has a dog
9. Ukrainian drinks tea
10. The Japanese smokes cravens
11. The old golds smoker has a snail
12. The gitanes smoker drinks wine
13. The chesterfields smoker's neighbor has a fox
14. The kools smoker's neighbor has a horse

Who drinks water?
Who owns the zebra?

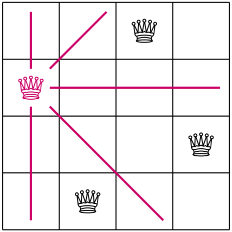
Zebra

- $\mathcal{V} = \{m_{blue}, m_{yellow}, m_{orange}, m_{red}, m_{green}, m_{English}, m_{Spanish}, m_{Japanese}, m_{Norwegian}, m_{Ukrainian}, m_{dog}, m_{horse}, m_{snail}, m_{fox}, m_{zebra}, m_{coffee}, m_{water}, m_{milk}, m_{tea}, m_{wine}, m_{chesterfields}, m_{cravens}, m_{gitanes}, m_{kools}, m_{oldgolds}\}$
- $\forall v \in \mathcal{V},\ D_v = \{1, 2, 3, 4, 5\}$

N-queens

Description

- On a $n \times n$ chessboard
- Place n queens so that no queen can capture another one

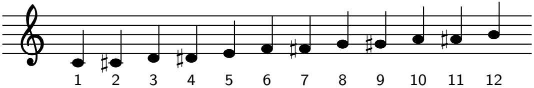


Music: all-interval series

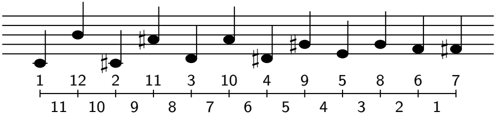
Description

- In the 1920s, Arnold Schönberg created a compositional principle: dodecaphony
- Consider the chromatic scale, and look for a motif in which

  notes appear exactly once 0ntervals (between 2 successive notes) must be different



Example (Trivial solution)



Magic Square

Description

- Place all the numbers from 1 to n^2 on an $n \times n$ square
- The sum of each row, each column, and both diagonals must be equal

	17	24	1	8	15	→ 65
	23	5	7	14	16	→ 65
	4	6	13	20	22	→ 65
	10	12	19	21	3	→ 65
	11	18	25	2	9	→ 65
↙ 65	↓ 65	↓ 65	↓ 65	↓ 65	↓ 65	↘ 65

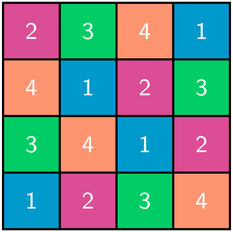
Latin square

Description

Given n colors, a Latin square is an $n \times n$ colored square such that:

- all cells are colored,
- each color appears exactly once in each row,
- each color appears exactly once in each column

Example (Solution for $n = 4$)



2	3	4	1
4	1	2	3
3	4	1	2
1	2	3	4